SADLER MATHEMATICS METHODS UNIT 1

WORKED SOLUTIONS

Chapter 3 Function

Exercise 3A

Question 1

a one to one ∴function

b one to many ∴not a function

c many to one ∴ function

d one to many ∴not a function

e many to one ∴function

f one to many ∴not a function

Question 2

a function

b function

c not a function (fails vertical line test)

d nota function (fails vertical line test)

e function

f not a function (fails vertical line test)

Question 3

a $1 \times 2 + 3 = 5$

$2 \times 2 + 3 = 7$

$3 \times 2 + 3 = 9$

$4 \times 2 + 3 = 11$

$\{5, 7, 9, 11\}$

b $(1 + 3) \times 2 = 8$

$(2 + 3) \times 2 = 10$

$(3 + 3) \times 2 = 12$

$(4 + 3) \times 2 = 14$

$\{8, 10, 12, 14\}$

c $1 \div 1 = 1$

$2 \div 2 = 1$

$3 \div 3 = 1$

$4 \div 4 = 1$

$\{1\}$

d $\{y \in R, y \geq 0\}$

Question 4

a $f(4) = 5(4) - 2 = 18$

b $f(-1) = 5(-1) - 2 = -7$

c $f(3) = 5(3) - 2 = 13$

d $f(1.2) = 5(1.2) - 2 = 4$

e $f(3) + f(2) = 5(3) - 2 + 5(2) - 2 = 21$

f $f(5) = 5(5) - 2 = 23$

g $f(-5) = 5(-5) - 2 = -27$

h $f(a) = 5(a) - 2 = 5a - 2$

i $f(2a) = 5(2a) - 2 = 10a - 2$

j $f(a^2) = 5(a^2) - 2 = 5a^2 - 2$

k $3f(2) = 3[5(2) - 2] = 24$

l $f(a + b) = 5(a + b) - 2 = 5a + 5b - 2$

m $f(p) = 5p - 2 = 33$

$5p = 35$

$p = 7$

n $f(q) = 5q - 2 = -12$

$5q = -10$

$q = -2$

Question 5

a $f(4) = 4(4) - 7 = 9$

b $f(0) = 4(0) - 7 = -7$

c $g(3) = 3^2 - 12 = -3$

d $g(-3) = (-3)^2 - 12 = -3$

e $h(-5) = (-5)^2 - 3(-5) + 3 = 43$

f $h(5) = (5)^2 - 3(5) + 3 = 13$

g $h(-2) = (-2)^2 - 3(-2) + 3 = 13$

h $3f(a) = 3(4a - 7) = 12a - 21$

i $f(3a) = 4(3a) - 7 = 12a - 7$

j $3g(a) = 3[a^2 - 12] = 3a^2 - 36$

k $g(3a) = (3a)^2 - 12 = 9a^2 - 12$

l $g(p) = p^2 - 12 = 24$

$p^2 = 36$

$p = \pm 6$

m $g(q) = h(q)$

$q^2 - 12 = q^2 - 3q + 3$

$3q = 15$

$q = 5$

n $h(r) = f(r) + 28$

$r^2 - 3r + 3 = 4r - 7 + 28$

$r^2 - 7r - 18 = 0$

$(r - 9)(r + 2) = 0$

$r = -2, 9$

Question 6

a $x-1 \geq 0$

$x \geq 1 \therefore f(x)$ is not defined for $x < 1$

b All values of x are possible, no exclusions required

c $\frac{1}{x}$ is not defined for $x = 0$

d $\frac{1}{1-x}$ is not defined when the denominator is zero

$1 - x \neq 0 \Rightarrow x \neq 1$

Question 7

a $f(x) = \sqrt{x-1}$ cannot produce values less than 0.

b $f(x) = x^2 + 1$ cannot produce values less than 1 as x^2 has a minimum value of 0.

c $f(x) = \frac{1}{x}$ cannot produce a value of 0.

d $f(x) = \frac{1}{1-x}$ cannot produce a value of 0.

Question 8

$f(0) = 5, f(3) = 8$

$\text{Range}: \{y \in \mathbb{R} : 5 \leq y \leq 8\}$

Question 9

$f(0) = -3, f(3) = 0$

$\text{Range}: \{y \in \mathbb{R} : -3 \leq y \leq 0\}$

Question 10

$f(-2) = -6,\ f(5) = 15$
Range: $\{y \in \mathbb{R} : -6 \leq y \leq 15\}$

Question 11

$f(5) = 20,\ f(10) = 40$
Range: $\{y \in \mathbb{R} : 20 \leq y \leq 40\}$

Question 12

$f(0) = -1,\ f(5) = 9$
Range: $\{y \in \mathbb{R} : -1 \leq y \leq 9\}$

Question 13

$f(0) = 1,\ f(5) = -4$
Range: $\{y \in \mathbb{R} : -4 \leq y \leq 1\}$

Question 14

$f(-1) = 1,\ f(3) = 9$ but $f(0) = 0$
Range: $\{y \in \mathbb{R} : 0 \leq y \leq 9\}$

Question 15

$f(-2) = 1,\ f(3) = 16$ but $f(-1) = 0$
Range: $\{y \in \mathbb{R} : 0 \leq y \leq 16\}$

Question 16

$f(-1) = 2,\ f(3) = 10$ but $f(0) = 1$
Range: $\{y \in \mathbb{R} : 1 \leq y \leq 10\}$

Question 17

$f(1) = 1,\ f(4) = 0.25$
Range: $\{y \in \mathbb{R} : 0.25 \leq y \leq 1\}$

Question 18

$f(1) = 1$, as $x \to 0, \dfrac{1}{x} \to \infty$
Range: $\{y \in \mathbb{R} : y \geq 1\}$

Question 19

minimum value : $f(0) = -1$
Range: $\{y \in \mathbb{R} : y \geq -1\}$

Question 20

minimum value : $f(0) = 4$
Range: $\{y \in \mathbb{R} : y \geq 4\}$

Question 21

Range: $\{y \in \mathbb{R} : y \neq 0\}$

Question 22

$f(0) = -1$ but $f(x) \neq 1$ (Try solving $f(x) = 1$)
Range: $\{y \in \mathbb{R} : y \neq 1\}$

Question 23

One to one function

$f(3) = 3,\ f(4) = 4$ and so on

Question 24

One to one as domain is limited to positive values

Question 25

Many to one

$f(-2) = f(2) = 4$

Question 26

Many to one

$f(x) = f(-x) = x^2$

Question 27

One to one

Each *y* value has only one square root due to the restriction of domain

Question 28

One to one

Each y value has only one square root due to the use of the radical sign

Question 29

$f(x) = 2x + 3$
Domain : $\{x : x \in \mathbb{R}\}$ or $\mathbb{R}$
Range : $\{y : y \in \mathbb{R}\}$ or $\mathbb{R}$

Question 30

$f(x) = x^2$
Domain : $\{x : x \in \mathbb{R}\}$ or $\mathbb{R}$
Range : $\{y : y \in \mathbb{R}, y \geq 0\}$

Question 31

$f(x) = \sqrt{x}$
Domain : $\{x : x \in \mathbb{R}, x \geq 0\}$
Range : $\{y : y \in \mathbb{R}, y \geq 0\}$

Question 32

$f(x) = \sqrt{x-3}$
$x - 3 \geq 0 \Rightarrow x \geq 3$
Domain : $\{x : x \in \mathbb{R}, x \geq 3\}$
Range : $\{y : y \in \mathbb{R}, y \geq 0\}$

Question 33

$f(x) = \sqrt{x+3}$
$x + 3 \geq 0 \Rightarrow x \geq -3$
Domain : $\{x : x \in \mathbb{R}, x \geq -3\}$
Range : $\{y : y \in \mathbb{R}, y \geq 0\}$

Question 34

$f(x) = 5 + \sqrt{x-3}$

$x - 3 \geq 0 \Rightarrow x \geq 3$

Domain : $\{x : x \in \mathbb{R}.x \geq 3\}$

Range : $\{y : y \in \mathbb{R}, y \geq 5\}$

Question 35

$f(x) = \dfrac{1}{x-3}$

$x - 3 \neq 0 \Rightarrow x \neq 3$

Domain : $\{x : x \in \mathbb{R}, x \neq 3\}$

Range : $\{y : y \in \mathbb{R}, y \neq 0\}$

Question 36

$f(x) = \dfrac{1}{\sqrt{x-3}}$

$x - 3 > 0 \Rightarrow x > 3$

Domain : $\{x : x \in \mathbb{R}, x > 3\}$

Range : $\{y : y \in \mathbb{R}, y > 0\}$ as the denominator may only take postive values

Question 1

a

$$\frac{2x-1}{3}=\frac{3x+2}{5}$$
$$5(2x-1)=3(3x+2)$$
$$10x-5=9x+6$$
$$x=11$$

b

$$\frac{3x-1}{2}+7=\frac{2x+7}{3}$$
$$6\times\left[\frac{3x-1}{2}\right]+6\times 7=6\times\left[\frac{2x+7}{3}\right]$$
$$3(3x-1)+42=2(2x+7)$$
$$9x-3+42=4x+14$$
$$9x+39=4x+14$$
$$5x=-25$$
$$x=-5$$

Question 2

$f(1)=3-2(1)=1$
$f(2)=3-2(2)=-1$
$f(3)=3-2(3)=-3$
$f(4)=3-2(4)=-5$
$\therefore$ range $\{-5,-3,-1,1\}$

Question 3

Graph 1 range: $\{y\in\mathbb{R}:-1\le y\le 4\}$

Graph 2 range: $\{-1,0,1,2,3,4\}$

Question 4

a $(a+b)^2 = a^2 + 2ab + b^2$

b $$\begin{aligned}(a+b)^3 &= (a+b)(a+b)^2 \\ &= (a+b)(a^2+2ab+b^2) \\ &= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3\end{aligned}$$

c $$\begin{aligned}(a+2b)^3 &= (a+2b)(a+2b)^2 \\ &= (a+2b)(a^2+4ab+4b^2) \\ &= a^3 + 4a^2b + 4ab^2 + 2a^2b + 8ab^2 + 8b^3 \\ &= a^3 + 6a^2b + 12ab^2 + 8b^3\end{aligned}$$

d $$\begin{aligned}(a-2b)^3 &= (a-2b)(a-2b)^2 \\ &= (a-2b)(a^2-4ab+4b^2) \\ &= a^3 - 4a^2b + 4ab^2 - 2a^2b + 8ab^2 - 8b^3 \\ &= a^3 - 6a^2b + 12ab^2 - 8b^3\end{aligned}$$

Question 5

a Function – passes vertical line test

One to one – passes horizontal line test

b Function - passes vertical line test

Many to one – fails horizontal line test

c Not a function – fails vertical line test

d Function – passes vertical line test

Many to one

e Function – passes vertical line test

One to one – passes horizontal line test

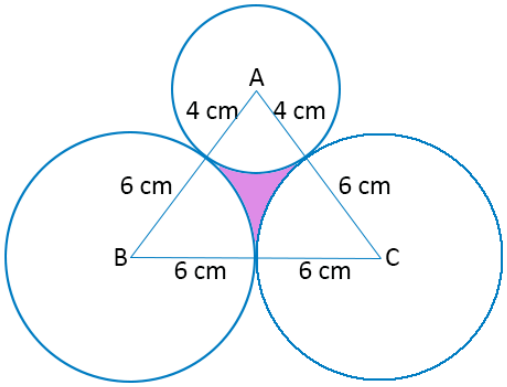
f Not a function

Question 6

$$\cos\angle ABC = \frac{10^2 + 12^2 - 10^2}{2 \times 10 \times 12}$$

$$\angle ABC = 0.927$$

$$\therefore \angle ACB = 0.927 \ \& \ \angle BAC = 1.288$$



Area of triangle ABC : $0.5 \times 10 \times 12 \times \sin 0.927 = 48.0$

Area of sector in circle centre A : $0.5 \times 4^2 \times 1.288 = 10.3$

Area of sector in circle centre B : $0.5 \times 6^2 \times 0.927 = 16.7$

Area of triangle outside of circle (shaded pink): $48.0 - (10.3 + 16.7 \times 2) = 4.3\,\text{cm}^2$

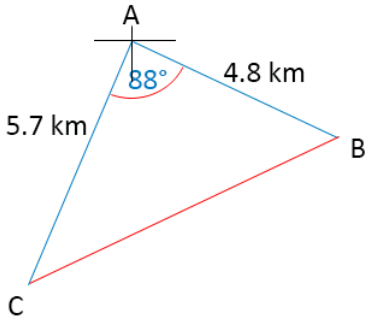
Question 7

$$BC^2 = 4.8^2 + 5.7^2 - 2 \times 4.8 \times 5.7 \times \cos 88°$$

$$BC = 7.3\,\text{km}$$

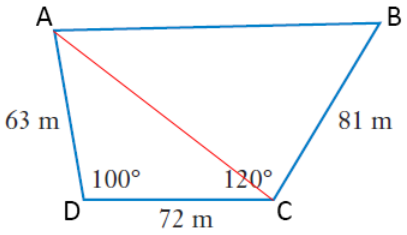
$$\frac{\sin\angle ACB}{4.8} = \frac{\sin 88°}{7.3}$$

$$\angle ACB = 41°$$



B is 7.3 km away from C on a bearing of 064°

Question 8



Area of ΔADC

$= 0.5 \times 63 \times 72 \times \sin 100°$

$= 2233.5\,\text{m}^2$

$AC^2 = 63^2 + 72^2 - 2 \times 63 \times 72 \times \cos 100°$

$AC = 103.6$ m

$$\frac{\sin \angle CAD}{72} = \frac{\sin 100°}{103.6}$$

$\angle CAD = 43°$

$\angle ACD = 180 - 43 - 100 = 37°$

$\angle ACB = 120 - 37 = 83°$

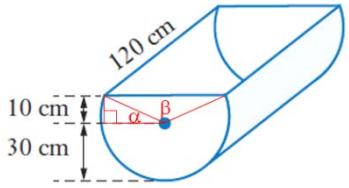
Area of ΔACB

$= 0.5 \times 103.6 \times 81 \times \sin 83°$

$= 4164.5\,\text{m}^2$

Total area $= 6398\,\text{m}^2$

Question 9



$$\sin \alpha = \frac{10}{30}$$

$\alpha = 0.34$

$\beta = \pi - 2 \times 0.34 = 2.46$

Area of segment missing from front face

$0.5 \times 30^2 (2.46 - \sin(2.46))$

$= 823.5\,\text{cm}^2$

Volume of trough

$(30^2 \pi - 823.5) \times 120$

$= 240\ 474\,\text{cm}^3$

Capacity of trough

$240474\,\text{cm}^3$ will hold 240471 mL=240.474L

Capacity is 240 L (nearest L)